CHAPTER 5

REACTOR DYNAMICS

The neutron population in a nuclear reactor may change with time for a number of reasons: nuclear fuel shuffling, control rod motion, fuel burnup, coolant flow perturbations, to name but some. Since a change in neutron density has an immediate effect on the power density it is necessary that both local and global temporal effects be assessed. In this chapter we will discuss some of the dynamic characteristics of a nuclear reactor in general and, where appropriate, refer to the CANDU reactor in particular. The emphasis will be on the more basic and dominant characteristics of temporal effects.

5.1 SIMPLIFIED POINT KINETICS ANALYSIS

As a general introductory example we consider the case of monoenergetic neutrons which appear in the reactor as a result of fission and after an average life time are absorbed. We further assume that we can consider our system to be such that the time behaviour is the same everywhere in the reactor core; hence we may consider neutron densities which vary only with time. The relationship between the number of neutrons at time t, $n(t)$, and the number of neutrons an average-neutron life time later, $n(t + \ell)$, is obtained by the inclusion of the rate of change of neutron density for the time interval ℓ

$$n(t + \ell) = n(t) + \ell \frac{dn(t)}{dt} \quad . \tag{5.1}$$

Recalling our definition of the neutron multiplication constant k_∞ permits us to write for this case

$$k_\infty = \frac{n(t + \ell)}{n(t)} \quad . \tag{5.2}$$

By substitution in Eq. (5.1) we obtain

$$k_\infty n(t) = n(t) + \ell \frac{dn(t)}{dt} \quad , \tag{5.3}$$

or

$$\frac{dn(t)}{dt} = \left(\frac{k_\infty - 1}{\ell}\right) n(t) \quad . \tag{5.4}$$

Integrating this first order partial differential equation to obtain an explicit expression for the time dependence of the neutron density yields

$$n(t) = n_0 \exp[(\frac{k_\infty - 1}{\ell})t] \ , \tag{5.5}$$

where n_0 is the initial neutron concentration. Since the neutron density is directly proportional to the reactor power density, we obtain an equivalent exponential time dependence for the power density:

$$p(t) = p_0 \exp[(\frac{\Delta k_\infty}{\ell})t] \ , \tag{5.6}$$

where

$$\Delta k_\infty = k_\infty - 1 \ . \tag{5.7}$$

This last term is generally called the "excess neutron multiplication".

Thus we note that whether the reactor power increases exponentially, is steady, or decreases exponentially, depends entirely upon whether the neutron multiplication constant k_∞ is greater than one, exactly equal to one, or less than one. One method of changing the neutron multiplication is to change the control rod position. We illustrate this in Fig. 5.1 for a simplified one-control-rod nuclear reactor.

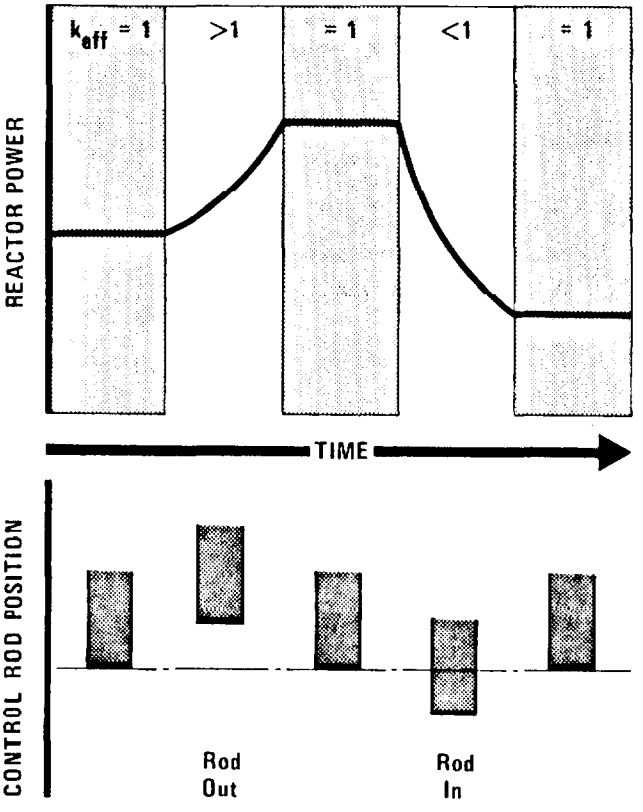


FIG. 5.1: Relationship between reactor power and control rod position.

It is informative to consider a numerical example. As we indicated in Chapter 4, the neutron life time is of the order of 0.001 sec. Supposing that conditions are such that during each successive neutron generation the neutron population increases by as little as 1/2% yielding $\Delta k_\infty = 0.005$. During one second therefore, the reactor power increase - if left uncontrolled - would be given by

$$p(t) = p_0 \exp[(\frac{0.005}{0.001})t] = p_0 \times 148 \ . \tag{5.8}$$

Thus, for such small changes in neutron multiplication, the reactor power would increase by a factor of almost 150 during one second. Indeed, if this were so it would be extremely difficult, if not impossible, to control a power reactor. However, in the above simplified analysis we have purposely neglected the important role of delayed neutrons. We now turn to a discussion of this phenomenon.

5.2 DELAYED NEUTRONS

In Chapter 2 we discussed radioactivity and referred to some of the radiations emitted by radioactive nuclei. From a reactor control point of view, it is significant that some of the radioactive fission products are neutron rich and eject neutrons. These delayed neutron precursers possess various half lives and thus contribute to the appearance of neutrons in the reactor at various time after the originating fission event has occurred. Because these delay times are relatively long when compared to the prompt neutron life time they have an effect far out of proportion to their numbers; indeed, the fraction of delayed neutrons is less than 1%. In Table 5.1 we list the fractions of delayed neutrons grouped according to their average delay times after fission, ℓ_i. Note that the delayed neutron contribution from Uranium-235 fission is 0.642%. The contribution of delayed neutrons due to photofission processes, that is fission induced by gamma rays, has not been included; its contribution is about 0.04%.

Isotope	Delayed Neutron Group i	Fraction of Delayed Neutron β_i(%)	Mean-time after fission ℓ_i(sec)
^{235}U	1	0.024	78.7
	2	0.137	31.5
	3	0.121	8.69
	4	0.261	3.21
	5	0.082	0.714
	6	0.017	0.258
	Total(%)	0.642	

TABLE 5.1: Delayed neutron data associated with fission of Uranium-235.

With this information we consider a reevaluation of the numerical example discussed in the preceding section. It seems plausible to consider an effective value of the neutron life time ℓ in Eq. (5.6) which incorporates the contribution of the delayed neutrons. We therefore write the effective neutron life time as composed of the two components associated with prompt and delayed neutrons:

$$\ell_{eff} = \ell_{\substack{prompt \\ neutron}} + \ell_{\substack{delayed \\ neutron}} \quad . \tag{5.9}$$

Using the known values of life times weighted by the prompt and delayed neutron fractions, suggests that we use

$$\ell_{eff} = (1 - 0.0064)\ell + \sum_{i=1}^{6} \beta_i \ell_i$$

$$= 0.0010 + 0.0189 + 0.0432 + 0.0105 + 0.0084 + 0.0006 + 0.0001$$

$$= 0.0827 \quad . \tag{5.10}$$

Here we have used the data for β_i and ℓ_i as listed in Table 5.1.

For the same neutron excess neutron multiplication, $\Delta k = 0.005$, the power increase during one second is now given by

$$p(t) = p_o \exp[(\frac{0.0050}{0.0827})t] = p_o \times 1.062 \quad . \tag{5.11}$$

Thus, the reactor power has in fact increased by only 6.2% in one second rather than the 14,700% increase calculation previously, Eq. (5.8). Such power changes during one second can of course be readily controlled using current control systems.

The above comparative calculation is illustrative of a fundamental feature of reactor control: a nuclear reactor is operated so as to be subcritical with respect to prompt neutrons alone. Since the delayed neutrons have time constants of the same order as those of available control systems, the reactor is, in effect, controlled by the delayed neutrons.

Note that Eq. (5.4) is an exceedingly simple description of the time dependence of the neutron population; indeed we recognize that it only pertains to the prompt neutrons only and not to the delayed neutron contribution. We indicate here, rather descriptively, how this equation can be modified to account for the delayed neutron contribution. First we correct the term on the right hand side for the delayed neutron fraction and then add the delayed neutron contribution rate to write

$$\frac{dn(t)}{dt} = [(\frac{k_\infty - 1}{\ell}) - (\frac{\beta}{\ell})]n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \quad , \tag{5.12}$$

where β is the total delayed neutron fraction, C_i represents the neutron precurser density for the i'th group of delayed neutrons and λ_i is the decay constant for this group. Note that each term $\lambda_i C_i$ represents - according to our analysis in Chapter 2 - the decay rate of a radioactive nucleus which is a neutron precurser.

The precurser density clearly varies with the neutron density; the more fissions occur the greater will be the precurser density. At the same time, since the neutron precursers are radioactive, they will decay. For the i'th precurser group we therefore write an equation which describes this balance between the production and decay of these neutron precursers,

$$\frac{dC_i(t)}{dt} = \text{production} - \text{decay} = \beta_i \frac{n(t)}{\ell} - \lambda_i C_i(t) \ . \qquad (5.13)$$

Although there are numerous delayed neutron precursers, it has been found that most can be combined to yield six distinct groups yielding i = 1,2,...,6. Thus, in a point kinetics analysis, a system of seven equations would need to be solved simultaneously, Eq. (5.12) and Eq. (5.13):

$$\frac{dn(t)}{dt} = \left(\frac{\Delta k - \beta}{\ell}\right) n(t) + \sum_{i=1}^{6} \lambda_i C_i \ . \qquad (5.14)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\ell} n(t) - \lambda_i C_i(t). \qquad i = 1,2,...,6 \qquad (5.15)$$

We will not discuss the detailed solution to this system of equations but indicate that the solution thus obtained bears out the general correctness of our previous simplified analysis. This is shown in Fig. 5.2 which is based on parameters appropriate to the CANDU reactor.

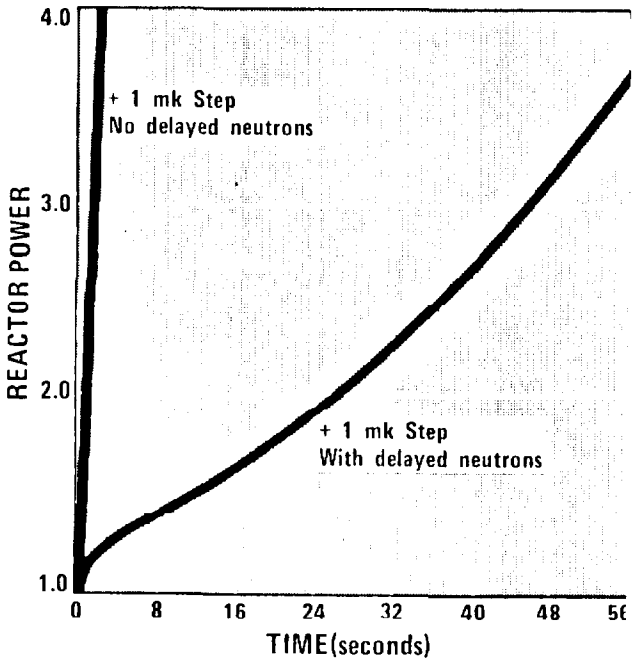


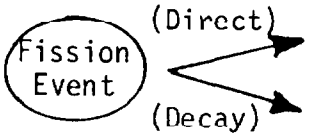
FIG. 5.2: Comparison between a power excursion without and wtih delayed neutrons; in both cases, the same excess neutron multiplication, $\Delta k = 0.001$ is considered.

5.3 XENON POISONING

In the discussion of the fission process and radioactive decay, we referred to various fission products which may exist in the reactor core. One of these isotopes, Xenon-135, has an unusual effect on reactor behaviour; this effect is attributable to its very large neutron absorption cross section of $\sigma_a \simeq 3.6 \times 10^6$ barns at thermal neutron energies and its half-life of $T_{1/2} = 9.2$ hours.

We note that the 3.6×10^6 barns microscopic absorption cross section of Xenon-135 is substantially larger than the 6.8×10^2 barns absorption cross section of the fissile nucleus Uranium-235 corresponding to the same energy. Therefore, even if the concentration of the Xenon-135 isotope is small, a relatively large number of neutrons would be removed from circulation by absorption in this non-fissile nucleus. This parasitic neutron removal by Xenon-135 is called xenon poisoning.

Consider a closer examination of the processes and mathematical description involving this isotope. Xenon appears either as a result of the fission process as a direct fission product and as a result of the decay chain initiated by the fission product Tellurium-135. We may symbolically represent this process by



$$^{135}Xe \rightarrow {}^{135}Cs \rightarrow {}^{135}Ba(\text{stable}) \qquad (5.16)$$

$$^{135}Te \rightarrow {}^{135}I \rightarrow {}^{135}Xe \rightarrow {}^{135}Cs \rightarrow {}^{135}Ba(\text{stable})$$

The total yield of Xenon-135 is 0.2% by the direct fission yield process and 6.1% by the decay chain process. Thus the radioactive decay chain process dominates. The decay processes all involve beta emission and half-lives which are given in the following:

$$^{135}Te \xrightarrow[T_{1/2}<0.5m]{\beta^-} {}^{135}I \xrightarrow[T_{1/2}=6.7h]{\beta^-} {}^{135}Xe \xrightarrow[T_{1/2}=9.2h]{\beta^-} {}^{135}Cs \ . \qquad (5.17)$$

Since tellurium decays so rapidly to iodine we may assume that iodine is directly produced in fission and write the time dependence for iodine as the production rate minus the decay rate,

$$\frac{dN_I}{dt} = \gamma_I \Sigma_f \phi - \lambda_I N_I \ , \qquad (5.18)$$

where N_I is the isotopic concentration of Iodine-135, λ_I is its decay constant while γ_I is the fission yield fraction of iodine, $\Sigma_f \phi$ is the energy averaged fission density rate in the fuel.

A similar expression may be written for Xenon-135 including, however, the important process of xenon removal by neutron absorption:

$$\frac{dN_X}{dt} = \lambda_I N_I - \lambda_X N_X - \sigma_{aX} N_X \phi \ . \qquad (5.19)$$

Consider now the effect of Xenon-135 during start-up and after shut-down of the reactor. As the reactor starts up, Iodine-135 will be produced and, according to Eq. (5.16) and Eq. (5.19), will decay to produce Xenon-135. Thus, the increasing concentration of this isotope has a poisoning effect because it removes neutrons in the core. Neutron multiplication in the reactor, that is k_{eff}, tends to decrease. The extent of this effect can be shown to be given by

$$k_{eff} - 1 = \frac{\text{(Neutron absorption in Xenon-135)/time}}{\text{(Fission)/time}} = \frac{\sigma_{aX} N_X}{\sigma_f N} \ , \qquad (5.20)$$

where the symbols are self-explanatory; $k_{eff} - 1$ is the neutron multiplication excess as used previously, Eq. (5.7). Normally, the numerical values of $k_{eff} - 1$ are in the 1/1000 range and, hence, the name milli-k (mk) is assigned.

It is now obvious that when the poisoning effect due to neutron absorption by Xenon-135 increases, that a control mechanism must be instituted to overcome this reactivity loss. This can be attained by the insertion of more fissile material or by the removal of neutron absorbing control rods. Independently of this control rod operation, the Xenon-135 concentration builds up after reactor start-up and therefore represents a reactivity load which eventually levels off. This build-up of the xenon reactivity load is governed by the reactor power level. Fig. 5.3 shows a typical xenon reactivity load build-up graphically; note that an equilibrium xenon concentration is reached after some two days of operation of a CANDU reactor.

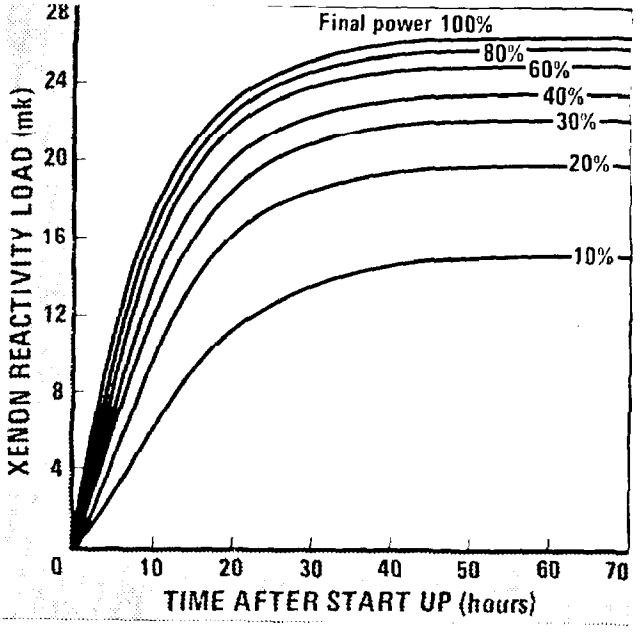


FIG. 5.4: Xenon-135 reactivity load after reactor start-up.

The final equilibrium concentration of Xenon-135 is readily calculated using Eq. (5.18) and Eq. (5.19). We set the time derivatives equal to zero and use the resultant algebraic equations to eliminate the iodine concentration, N_I, to obtain

$$N_{X,\infty} = \left(\frac{\gamma I}{\lambda_X - \sigma_{aX}}\right)\Sigma_f\phi \ . \qquad (5.21)$$

Note the importance of the fission rate and fission yield in the determination of the final xenon density.

Consider now the situation in which the reactor is suddenly shut down. Both Iodine-135 and Xenon-135 have attained an equilibrium concentration and represent a constant reactivity load. At shutdown, the production of Iodine-135 from fission stops suddenly; the existing concentration of this isotope decays with a half-life of 6.7 hours to yield Xenon-135. Now we note that, since the decay of Xenon-135 to Cesium-135 proceeds at a slower rate ($T_{1/2}$ = 9.2 hours), the build-up rate of xenon exceeds its decay rate. Hence, the Xenon-135 concentration increases and its reactivity load on the reactor increases as well. This process, however, cannot continue for too long because eventually the Iodine-135 becomes depleted and so will the xenon concentration. Thus the reactivity load reaches a maximum some 5 to 10 hrs after shutdown and then gradually decays; the extent of this effect in a CANDU reactor is determined by the initial power level, Fig. 5.4.

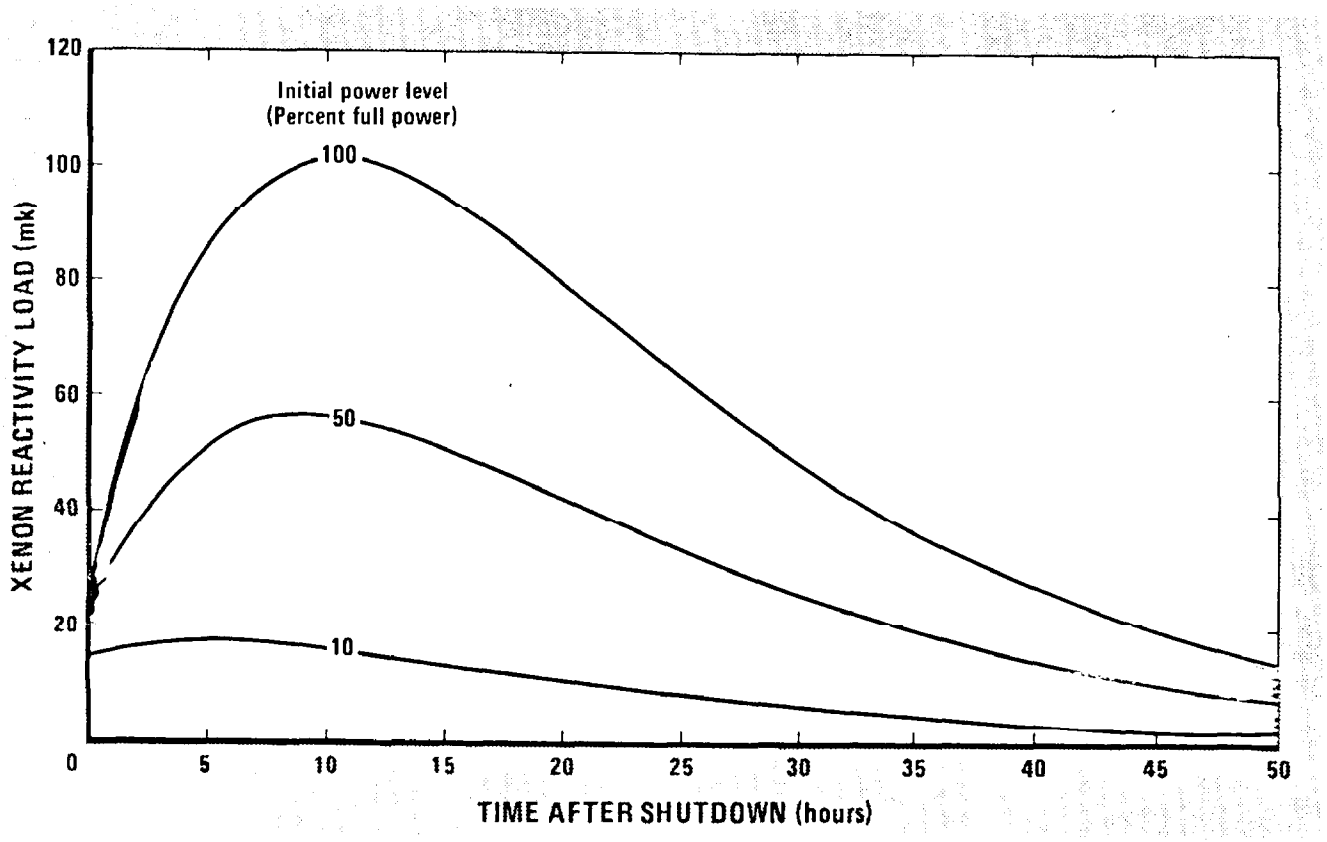


FIG. 5.4: Xenon-135 reactivity load following reactor shutdown.

5.4 REACTIVITY AND FEEDBACK

In the preceding section we discussed the effect of Xenon-135 on criticality of the nuclear reactor using the excess neutron multiplication, Δk,

$$\Delta k = k_{eff} - 1 \ , \tag{5.22}$$

as the describing parameter. It is common practice to normalize Δk with respect to k_{eff} and thus define the term reactivity by

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = \frac{\Delta k}{k_{eff}} \tag{5.23}$$

In general, k_{eff} is very close to unity so that numerically the reactivity ρ may be similar in magnitude to the excess neutron multiplication Δk.

As shown in the two preceding figures, the appearance of Xenon-135 can lead to significant parasitic neutron absorption and thus constitute a reactivity load on the reactor system. Adequate provisions in the form of built-in excess reactivity must be provided to compensate for this reactivity effect. Xenon-135, however, does not represent the only reactivity effect. There exist other fission products possessing a significant neutron absorption cross section. In addition, as the temperature of the core changes, a change also occurs in some of the neutron absorption cross sections. Clearly, the reactor control rods absorb neutrons and thus also represent a reactivity load.

The excess reactivity provided for in a nuclear reactor must be equal to the total reactivity load of the various reactivity effects enumerated above. It is convenient to think of these effects as a reactive balance. A typical balance for a CANDU reactor is shown in Fig. 5.5 where the total reactivity load adds up to 110 mk. Here we note the significant reactivity component associated with fresh fuel. This serves to illustrate the fact that the excess reactivity varies continually with time. The on-line refuelling capability of the CANDU reactor permits a measure of reactivity control since fuel bundles possessing a high reactivity load due to such factors as fission products can be removed and replaced with fresh fuel.

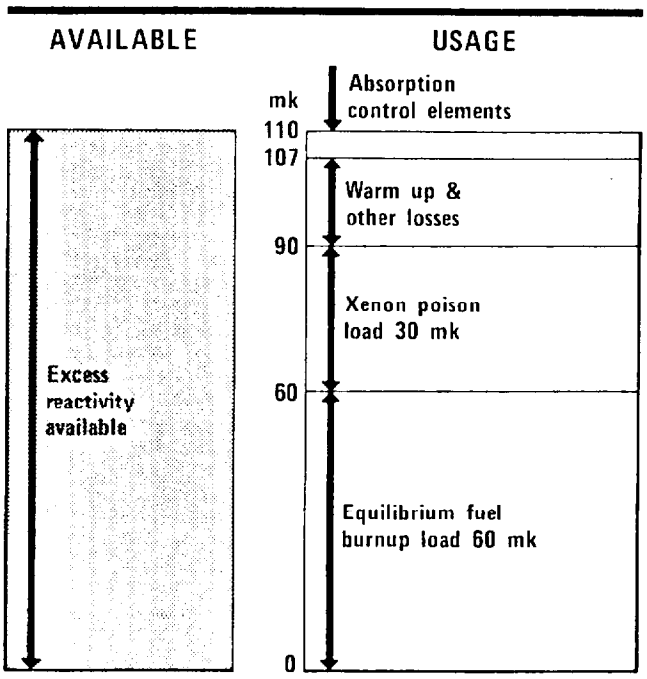


FIG. 5.5: Typical reactivity balance representation.

The nuclear reactor core is also characterized by certain feedback properties. For example, in Eq. (5.21) we note the dependence of the Xenon-135 equilibrium concentration on the flux and hence on the reactor power. Thus, as the reactor power level changes, a certain reactivity perturbation will be introduced eventually due to this fission product poison. Other effects are more immediate. For example, a change in power density will lead to a change in the local temperature. A change in temperature will instantaneously effect the magnitude of certain neutron cross sections and therefore affect the neutron density. This contribution may be termed a fuel temperature reactivity effect and can be compensated by appropriate control operation. As a third feedback effect we identify the role of coolant density changes and void formation which effects both heat transfer and the neutron spectrum. This too represents a feedback effect. A block diagram illustrating these feedback loops is illustrated in Fig. 5.6.

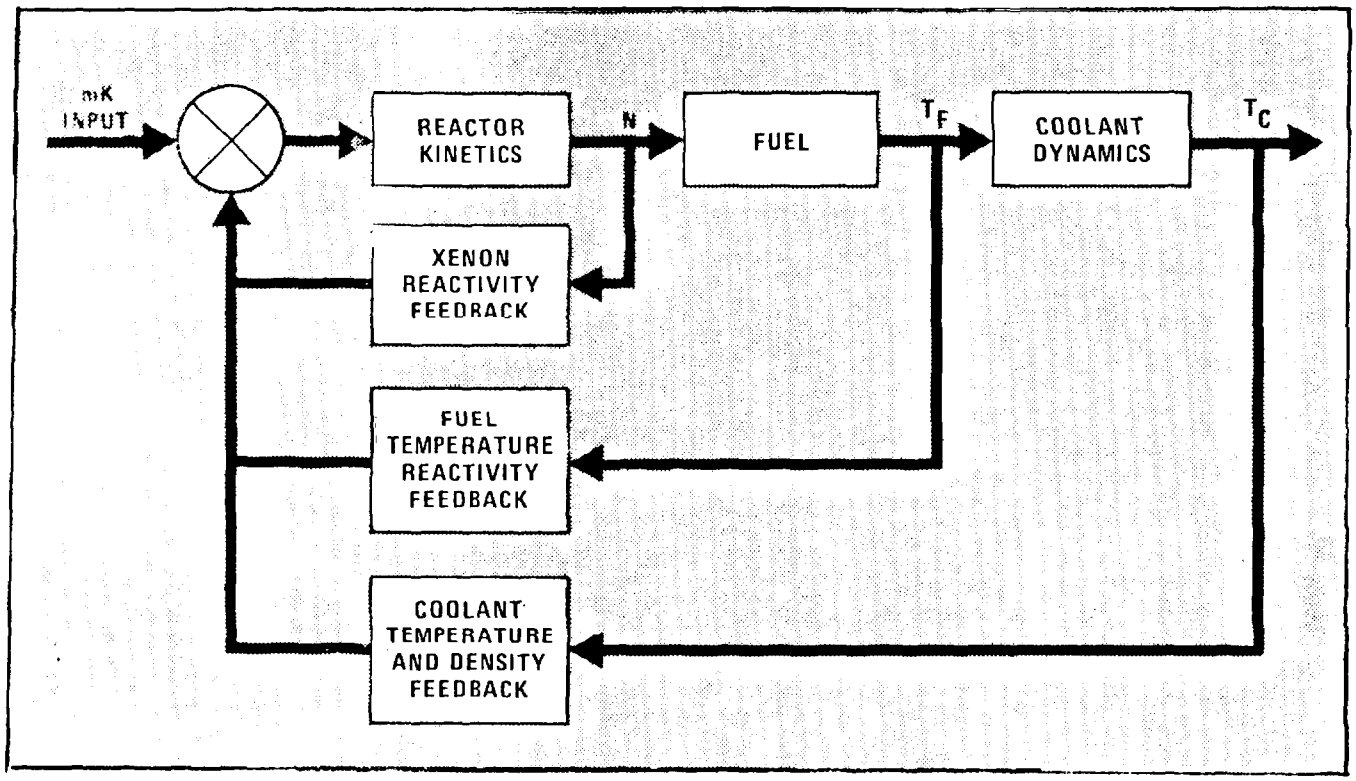


FIG. 5.6: Simplified feedback diagram for a nuclear reactor.